VP160 Recitation Class

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Approximation

Basic Formulas($x << 1, \theta << 1$)

1. $(1+x)^n \approx 1 + nx$
2. $sin(\theta) \approx \theta$
3. $cos(\theta) \approx 1 - \frac{1}{2}\theta^2 \approx 1$
4. $e^x \approx 1 + x$
5. $ln(x+1) \approx x$
6. Taylor Expansion:
   $$U(x) = U(x_0) + U'(x_0)(x - x_0) + \frac{1}{2}U''(x_0)(x - x_0)^2 + ...$$

Exercise($x << 1, \theta << 1$, m is a constant)

1. $\dfrac{1}{(2+x)^2}$
2. $1 - cos(\theta)$
3. $sin(0.1^o)$
4. $\dfrac{1}{(m-x)^2} - \dfrac{1}{(m+x)^2}$
5. $ln(\dfrac{m+x}{2m})$

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(What if $x << m$?)

Force and Inertial Frame of Reference

Force

Interaction between two objects or an object and the environment.

Inertial Frame of Reference

In a inertial frame of reference, if the net force on a particle is zero, then its acceleration is zero.

Newton's Law

1. A particle acted upon by zero net force moves with constant velocity.
2. In an inertial frame of reference, acceleration of a particle is directly proportional to the net force acting upon it, and inversely proportional to its mass.
3. The mutual forces of action and reaction between two bodies are equal in magnitude and opposite in direction.

Exercise 1

(1)Mass m hangs on a massless rope in a car moving with (a) constant velocity v, (b) constant acceleration a on a horizontal surface. What is the angle the rope forms with the vertical direction?

(2)What if the car slides (without friction) down a plane inclined at an angle α?

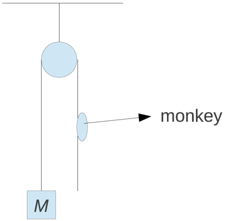
Exercise 2

A monkey with mass m holds a rope hanging over a frictionless pulley attached to mass M (see
gure). Discuss motion of the system if the monkey

(a) does not move with respect to the rope,

(b) climbs up the rope with constant velocity v_0 with respect to the rope,

(c) climbs up the rope with constant acceleration a_0 with respect to the rope.



Motion with Air/Fluid Drag

Consider a falling partical with linear drag and initial conditions $y(0) = 0, v_y(0) = 0$.

$$v_y(t) = \frac{mg}{k}(1 - e^{-\frac{k}{m}t})$$

$$y(t) = \frac{mg}{k}(t + \frac{m}{k}(e^{-\frac{k}{m}t} - 1))$$

How to derive the above formulas?

Simple Harmonic Oscillator

$$\ddot{x} + \frac{k}{m}x = 0$$

$$x = A cos(\omega_0 t) + B sin(\omega_0 t)$$

Question:

What if the equation is $\ddot{x} + \frac{k}{m}x = C$, where C is a constant?

Three methods to solve the Simple Harmonic Oscillation:

1. $F = -kx$ using Newton's law, then $T = 2\pi\sqrt{\dfrac{m}{k}}$
2. $\ddot{x} + \omega^2 x = C$ using Newton's law, then $T = \dfrac{2\pi}{\omega}$
3. $\ddot{x} + \omega^2 x = C$ using derivative of potential, then $T = \dfrac{2\pi}{\omega}$

Exercise 3

A particle falls on the Earth from a high altitude h. Neglecting air drag, find the time T when it hits the ground and the speed it has at this instant

Exercise 4.1

Discuss the motion of a particle that is placed on the inner surface of a spherical pot, close to its bottom, and released from hold (no friction).

Exercise 4.2

The same for a pot with cross-section in the shape of a cycloid placed upside-down

$$x = R(\gamma + sin\gamma), \quad y = R(1 - cos\gamma),$$

where $-\pi \leq \gamma \leq \pi$

Harmonic Oscillator with Linear Damping

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$$

1. Overdamped: $b^2 > 4km$
   $x(t) = C_1 e^{-(\frac{b}{2m}+\sqrt{\frac{b^2}{4m^2}-\omega_0^2})t} + C_2 e^{-(\frac{b}{2m}-\sqrt{\frac{b^2}{4m^2}-\omega_0^2})t}$
2. Critically damped: $b^2 = 4km$
   $x(t) = C_1 e^{-\frac{b}{2m}t} + C_2 t e^{-\frac{b}{2m}t}$
3. Underdamped: $b^2 < 4km$
   $x(t) = e^{-\frac{b}{2m}t}[A cos(\sqrt{\omega_0^2 - \frac{b^2}{4m^2}}t) + B sin(\sqrt{\omega_0^2 - \frac{b^2}{4m^2}}t)]$

Driven Oscillation

Equations

$$\ddot{x}(t) + \frac{b}{m}\dot{x} + \frac{k}{m}x = \frac{F_0}{m} cos \omega_{dr} t$$

$$A = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega_{dr}^2)^2 + (\frac{b\omega_{dr}}{m})^2}}$$

$$tan\phi = \frac{b\omega_{dr}}{m(\omega_{dr}^2 - \omega_0^2)}$$

$$\omega_{res} = \sqrt{\omega_0^2 - \frac{b^2}{2m^2}}$$

Reminders

1. The resonance frequency is lower than the natural frequency if we have drag.
2. The response of the system is not in phase with the drive.